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Antigravity for Aerospace Applications

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Administrative Note

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Foreword

Antigravity effects can be implemented by manipulating spacetime. This paper reviews several different theoretical approaches for exploring the possibility of controlling gravity by generating forces that counteract, or otherwise modify, gravity for the purpose of aerospace propulsion. Einstein's General Theory of Relativity is the theoretical framework guiding this study.

The paper also reviews other antigravity approaches via the interaction of quantum theory with gravitation. And it explores the question of which method or technique is best suited for aerospace applications and evaluates the make-or-break issues that limit them.

I. Introduction

Gravity is the bane of aerospace transportation. The force of the Earth's gravitational field acts to pull all objects, whether in motion or at rest, downward towards the Earth's surface. Because aerospace transportation involves the motion of vehicles through the atmosphere and/or into space, propulsion engineers are always faced with the requirement that aerospace vehicles will have to carry enough propellant and associated tankage in order to provide enough propulsive thrust to overcome the downward pull of gravity and achieve rectilinear motion. Energy has to be expended by a propulsion system to overcome the force of gravity in addition to providing for rectilinear motion, and the majority of propulsive energy is dedicated to overcome gravity. The aerospace propulsion engineer is faced with two choices for the control of gravity in this regard: passive control and active control. Modern aerospace propulsion technology, which is based on accumulated scientific knowledge since recorded history, can only achieve the passive control of gravity whereby a given propulsion device must develop a thrust that will passively counteract the Earth's gravitational pull, lift a vehicle off the surface, and propel it through the air or into space. Newton's laws of motion and gravity require that the fuel fraction of any aerospace vehicle can never be less than that given by a simple function of the ratio of the vehicle's maximum speed to the speed of its rocket plume, jet, fan, or propeller wake. For example, this limit implies that a single-stage rocket that accelerates to escape velocity must be composed of more than 93 percent fuel. That is because a rocket must accelerate its working fluid from rest (relative to the rocket) up to its exhaust speed. Thus, exhaust speeds for aircraft and chemical rockets are limited by material science, chemical reaction rates, and engineering factors to only a few thousand meters per second.

To date, there is no technology that can achieve the active control of gravity. If one could eliminate or otherwise control the Earth's gravity field, then one has the ability to dramatically reduce the amount of propellant, its tankage, and the overall structural size and mass of an aircraft or rocket because there will no longer be any need for these to overcome the pull of Earth's gravity while transporting a payload across the globe or into space. Instead, aerospace vehicles will only need to have the propellant mass and infrastructure necessary to change their kinetic energy from rest to a final velocity necessary to achieve atmospheric flight or space orbit. The Earth's gravitational well will no longer have any impact on aircraft, launch vehicle, or spaceflight dynamics if one were to achieve active gravity control. Aerospace vehicles would merely "levitate" in air and their propulsion systems would be optimized for change-in-velocity missions. However, it is possible to envision a form of active gravity control propulsion that would not require a change in kinetic energy.

One of the primary concepts for the goal of affecting gravity is "antigravity," which is a colloquial expression that specifically means the negation or repulsion of the force of gravity. A more general term that encompasses this notion and other possibilities is "gravity control."

If antigravity exists, it can be exploited to counteract or nullify the gravitational pull, or attraction, of a planetary (or stellar) body that acts upon a much smaller body. Einstein's General Theory of Relativity gives a prescription for a variety of different antigravity generators. Even Newton's law of gravity offers several different classical prescriptions. Newton's law of gravity can be used to simply nullify the gravity field of one body acting on another body by using a clever arrangement of masses. The theoretical possibility of antigravity also appears in quantum gravity theories, cosmological vacuum or dark energy, and quantum field theory. This report reviews all of these topics. The report will also review the topics of gravity control that include the production of antigravity (self-lifting) forces induced by quantum vacuum zero-point energy and by nonretarded quantum interatomic dispersion forces in a curved spacetime (that is, in a background gravitational field). The reader should bear in mind that many of these concepts are nowhere near having any form of practicable engineering implementation. However, the report will provide theoretical estimates to guide the way toward technological implementation of antigravity.

II. Concepts for Antigravity Within Newtonian Physics

The basic form of Newton's law of gravity is given by the standard expression for the gravitational force (F_{grav}) that mutually acts between two masses (Reference 1):

$$F_{\mathrm{grav}} = -\frac{G\, m_1\, m_2}{r^2} \tag{1}$$

where the negative sign indicates that F_{grav} is a (mutual) force of attraction, G is Newton's universal gravitation constant (6.673×10^{-11} Nm^2/kg^2), m_1 and m_2 are two interacting masses, and r is the radial distance between the two masses (note: MKS units are used throughout). Observe in Equation (1) that the force of gravity acting on a small test mass becomes stronger when the other (gravitating) mass is larger in magnitude or when the distance between them is very small, or both. Also recall that Equation (1) and Newton's second law of motion ($F = ma$) to define the magnitude of the gravitational acceleration a_g that acts on a small test mass m due to a larger (gravitating) mass M (Reference 1):

$$a_g = \frac{GM}{r^2} \tag{2}$$

If Earth is chosen to be the larger gravitating mass so that $M = M_{\oplus}$ (5.972×10^{24} kg), then according to Equation (2) a small test mass m placed near the Earth's surface, whereby $r \approx R_{\ominus}$ (6.378×10^{6} m), will experience a downward gravitational acceleration of $a_g \equiv g$ = 9.81 m/s^2.

NEGATING NEWTONIAN GRAVITY

It is possible to design an antigravity machine that can nullify Earth's gravity field using Newton's law of gravity. One way to use Equation (1) to nullify the Earth's gravitational pull at a particular location would be to locate another planet of equal mass above that location (Reference 2,3). The forces from the two Earth masses will cancel each other out over a broad region between them. Everything within this broad region will be in free fall. However, this is not a practical solution for aerospace flight since there is no way to manipulate and control another planetary sized body.

Along similar lines, Forward (Reference 2,3) suggested to consider using a ball of ultradense compact matter, corresponding to dwarf star or neutron star matter (~ 10^{11} - 10^{18} kg/m^3), having a diameter of 32 cm and a mass of 4 million metric tons. This ultradense ball will have a surface gravitational (attractive) force of 1-g. This small ultradense ball could be placed near the surface of the Earth and its 1-g gravity field will cancel the Earth's 1-g gravity field. All test objects placed in the broad region between the small ultradense ball and the Earth will thus be in free fall. Another option Forward (Reference 2-5) suggested would be to shape the compact ultradense matter into a disk that is 45 cm in diameter and 10 cm thick, and having the same mass and density as the small ultradense ball. Its gravitational acceleration is $a_g = 4G\rho\tau$, where ρ is the mass density of the disk and τ is its thickness. In this case, the disk will have a force of gravitational attraction that is the same on both sides, and it will be uniform near the center of the disk where the strength of the gravitational force will be 1-g. If this disk were to be placed very close above the Earth's surface, then there will be a gravitational force of 2-g above the disk (= 1-g due to the Earth's gravity field plus 1-g due to the top-side gravity field of the disk) while underneath the disk near its center there will be a gravity-free (or free fall) region because the Earth's gravity field underneath is canceled by the gravity field of the disk's bottom-side. While these are interesting antigravity machines, they are unfortunately not feasible from an engineering standpoint since one does not yet have the technology or means to create and handle ultradense compact matter.

ENERGY ESTIMATE FOR NEWTONIAN LEVITATION

An ideal propulsion breakthrough could take the form of the antigravity-based levitation of an aerospace vehicle within the Earth's atmosphere. Rockets like the Air Force DC-XA can hover above the ground for a time that is limited by the amount of rocket fuel available (Reference 6). But an ideal antigravity propulsion device should allow for the indefinite levitation of a vehicle above the Earth's surface. It is illustrative to estimate the energy required to levitate a 1-kg test mass above the Earth's surface. This will help quantify a potentially key engineering parameter for such a levitation system. A generic estimate can be found by considering the amount of energy per unit mass required to nullify the (magnitude) of the Earth's gravitational potential energy $\mathrm{E}_{\mathrm{lev}}$ for a test mass m hovering at height h above the Earth's surface:

$$\mathrm{E}_{\mathrm{lev}} = \frac{GM_{\oplus}m}{h} \quad (\mathrm{J/kg}) \tag{3}$$

Equation (3) can also be derived by calculating how much energy is required to completely remove a test mass from the Earth's surface to infinity. This calculation is more in line with the analogy to nullify the effect of gravitational energy. And Equation (3) also represents the energy required to stop a test mass at the levitation distance h if it were falling in from infinity with zero initial velocity.

Setting $h \approx R_{\oplus}$ and m = 1 kg in Equation (3), the result is $\mathrm{E}_{\mathrm{lev}}$ = 62.5 MJ/kg. This is 2.05 times the kinetic energy required to put the test mass into low Earth orbit (LED). However, this estimate will require some adjustment that depends upon the type of theory and its technological implementation. That is because the operational energetics of a putative antigravity propulsion system must be considered in conjunction with $\mathrm{E}_{\mathrm{lev}}$.

III. Concepts for Antigravity Within General Relativity

In the Sections that follow the known types of antigravity that can be derived from Einstein's General Theory of Relativity are described and summarized, which is the modern relativistic theory of gravity.

ANTIGRAVITY VIA GRAVITOMAGNETIC FORCES

Historical Foundations

Heaviside (Reference 7) (in 1883), Einstein (prior to the 1916 publication of his General Theory of Relativity), Thirring (Reference 8,9), and Thirring and Lense (Reference 10) (see also, Reference 11) showed that general relativity theory provides a number of ways to generate non-Newtonian gravitational forces via the splitting of gravitation into electric and magnetic field type components. These forces can be used to counteract the Earth's gravitational field, thus acting as a form of antigravity. General relativity theory predicts that a moving source of mass-energy can create forces on a test body which are similar to the usual centrifugal and Coriolis forces, although much smaller in magnitude. These forces create accelerations on a test body that are independent of the mass of the test body, and the forces are indistinguishable from the usual Newtonian gravitational force. The Earth's gravitational field can be counteracted by generating these forces in an upward direction at some spot on the Earth.

Forward (Reference 12) linearized Einstein's general relativistic field equation and developed a set of dynamic gravitational field relations similar to Maxwell's electromagnetic field relations. The resulting linearized gravitational field relations are a version of Newton's law of gravitation that obeys special relativity. The linearized gravitational field relations show that there is a unique correspondence between the gravitational field and the electric field. For example, the Newtonian gravitational field of an isolated mass is the gravitational analog to the electric field of an isolated electric charge.

Likewise, there is an analogy to a magnetic field contained within the linearized gravitational field relations. In Maxwellian electrodynamics, a magnetic field is due to the flow of an electric charge or an electric current. In other words, the electric field surrounding an electric charge in motion will appear as a magnetic field to stationary observers. If the observers move along with the charge, they see no relative motion, and so they will only observe the charge's electric field. Thus, the magnetic field is simply an electric field that is looked at in a moving frame of reference. In an analogous fashion, the linearized gravitational field relations show that if a (gravitational) mass is set into motion and forms a mass current, then a new type of gravitational field is created that has no source and no sink. This is called the Lense-Thirring effect, or rotational frame dragging effect, in which rotating bodies literally drag spacetime around themselves.

Forward's Dipole Gravitational Field Generator

Forward (Reference 13,14) used the linearized gravitational field relations plus aspects of the Lense-Thirring effect to develop models for generating antigravity forces. One example of an antigravity generator is based on a system of accelerated masses whose mass flow can be approximated by the electrical current flow in a wire-wound torus.

According to Maxwellian electrodynamics, an electric current flowing through a wire that is wrapped around a torus (or ring) causes a magnetic field to form inside the torus. If the current (I) in the wire increases with time, then the magnetic field **B** inside the torus also increases with time. This time-varying magnetic field in turn creates a dipole electric field E, as shown in Figure 1. The magnitude of the electric field at the center of the torus is given by:

$$\mathrm{E} = -\frac{\mu_0 N r^2 \dot{I}}{4\pi R_t^2} \tag{4}$$

where μ_0 is the vacuum electromagnetic permeability constant ($4\pi \times 10^{-7}$ H/m), N is the total number of turns of wire wound around the torus, $\dot{I}$ is the time rate-of-change of the electric current flowing through the wire, r is the radius of one of the loops of wire, and R_t is the radius of the torus.

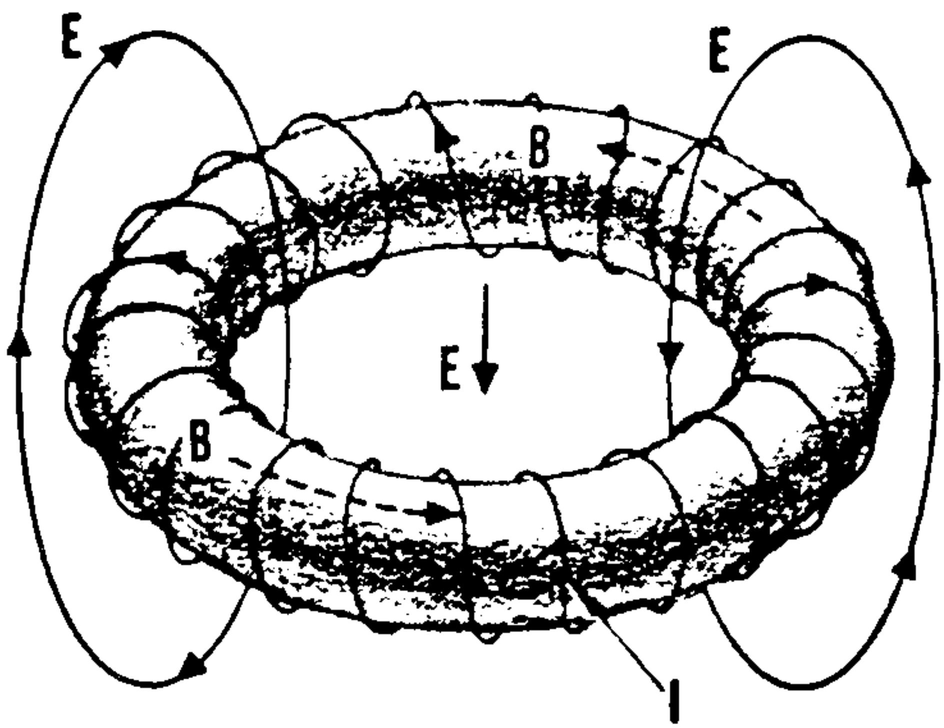

Figure 1. Dipole Electric Field Generator (Reference 14)

In a similar fashion, Forward's antigravity device is a dipole gravitational field generator. As shown in Figure 2, a mass flow T through a pipe wound around a torus induces a Lense-Thirring field **P** to form inside the torus. If the mass flow is accelerated, then the **P**-field increases with time, and thus a dipole gravitational field **G** is created. The magnitude of the anti-gravitational field at the center of the torus is given by:

$$\mathrm{G} = \frac{\eta_0 N r^2 \dot{T}}{4\pi R_t^2} \tag{5}$$

where η_0 is the vacuum "gravitational permeability" constant ($\equiv 16\pi G/c^2 = 3.73 \times 10^{-26}$ m/kg),[1] N is the total number of turns of pipe wound around the torus, $\dot{P}$ is the time rate-of-change of the mass current flowing through the pipe, r is the radius of one of the loops of pipe, R_t is the radius of the torus, and c is the speed of light (3×10^8 m/s) (Reference 14). One should note the striking similarity between Equations (4) and (5) for the dipole electric and dipole gravitational fields.

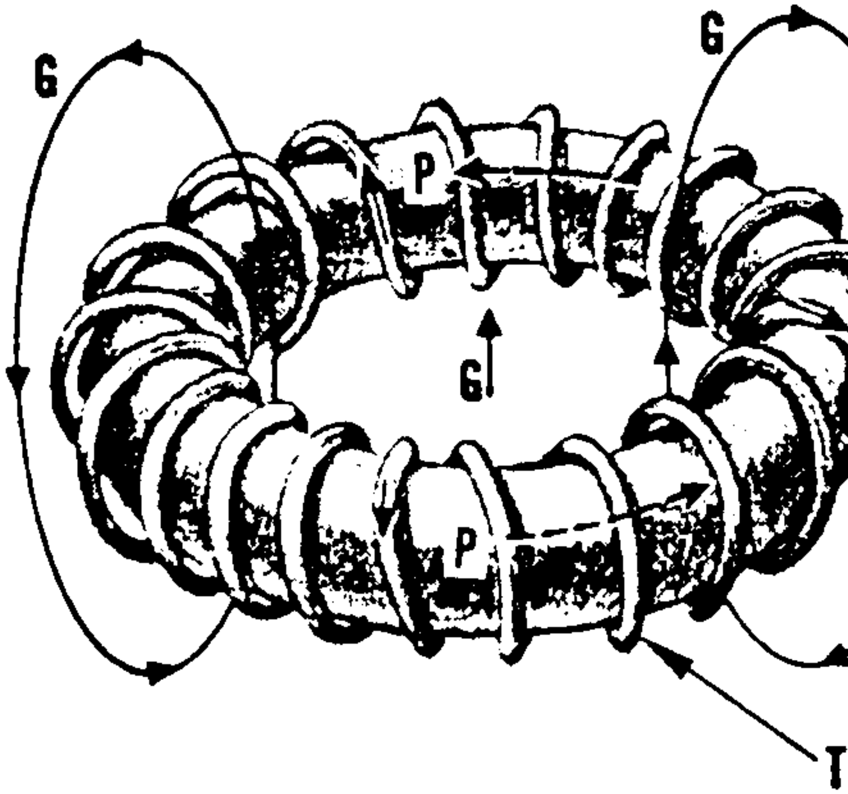

Figure 2. Diople Gravitational Field Generator (Reference 14)

Using Equation (5), Forward (Reference 13,14) showed that there would be a need to accelerate matter with the density of a dwarf star through pipes as wide as a football field wound around a torus with kilometer dimensions in order to produce an antigravity field (at the center of the torus) of **G** $\approx 10^{-10} a_{acc}$, where a_{acc} is the acceleration of the (dwarf star density) matter through the pipes. The tiny factor 10^{-10} is composed of the even smaller η_0, which is the reason why very large systems are required to obtain even a measurable amount of acceleration. To counteract the Earth's gravitational field of 1-g requires an antigravity field of 1-g (vectored upward), and thus the dwarf star density material within the pipes must achieve $a_{acc} = 10^{11}$ m/s^2 in order to accomplish this effect.

Forward (Reference 5) also identified a configuration comprised of a rotating torus of dense matter that turns inside-out like a smoke ring as another type of dipole gravitational field generator. As shown in Figure 3, an inside-out turning ring of very dense mass (M) will create an upward force (of acceleration a) in the direction of the (constant) mass motion (Mv, v is the mass velocity). This is also a feature of the

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[1] The vacuum "gravitational permittivity" constant is (Reference 12): $\gamma_0 \equiv (4\pi G)^{-1} = 1.19 \times 10^9$ kg·s^2/m^3.

Lense-Thirring effect. Forward's linearization analysis generalizes all of these effects into the following two key ingredients that are required to produce antigravity forces: 1) any mass with a velocity and an acceleration exerts many different general relativistic forces on a test mass, and 2) these forces act in the direction of the velocity and in the direction of the acceleration of the originating mass. In summary, these forces are equivalent to gravitational forces, which can be used to cancel the Earth's gravitational field.

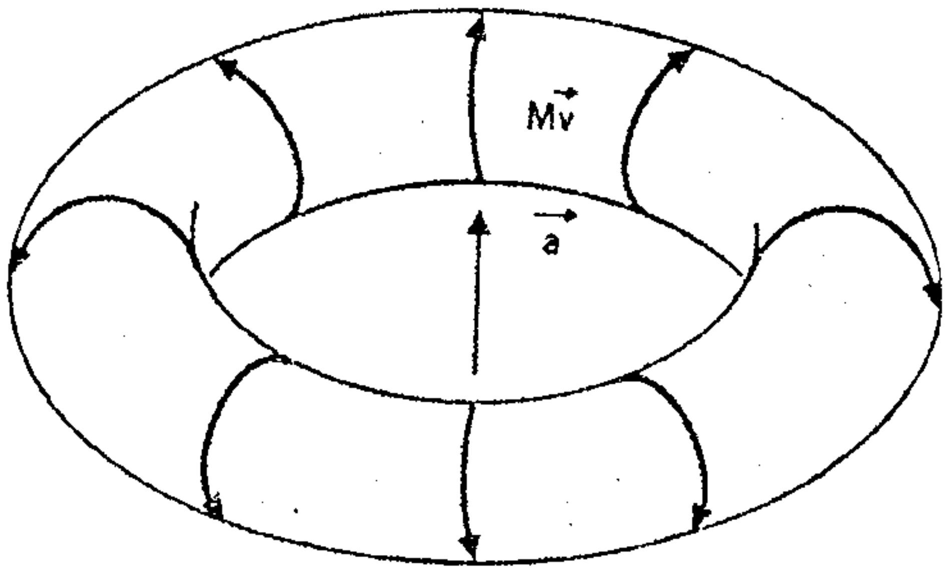

Figure 3. Dipole Gravitational Field Generator: Inside-Out Whirling Dense Matter Torus (Reference 5)

One can also view this genre of devices as a gravity catapult machine in which the machine pushes a body away using its general relativistic antigravity forces to impart a change in velocity. A space launch operator on the ground wanting to send a payload up into orbit would just ratchet up the strength of the (upward-directed) antigravity field to some value above 1-g, and after pressing the release button the payload accelerates up and away into orbit. These devices could also be placed in Earth orbit, stationed anywhere within the solar system, or even distributed throughout the galaxy in order to establish a network of gravity catapults. Space travelers could begin their trip by being launched from the catapult on the Earth's surface, and when they reach space they would jump through various catapults as needed to reach their destination.

FELBER'S RELATIVISTIC ANTIGRAVITY EFFECT

Felber (Reference 15) used the Schwarzschild solution of Einstein's general relativistic field equation to find the exact relativistic motion of a payload in the gravitational field of a mass moving with constant velocity. His analysis gives a relativistically exact (strong gravitational field condition) calculation showing that a mass, which radially approaches or recedes from a payload at a relative velocity of $v_{crit} > c/3^{1/2}$ ($v_{crit} \equiv$ critical velocity), will gravitationally repel the payload as seen by distant inertial observers. In other words, any source mass, no matter how large or small it is or how far away it is from a test body (payload), will produce an antigravity field when moving at any constant velocity above v_{crit}.

The exact relativistic strong-field condition that establishes the lower limit criterion for v_{crit} to induce antigravity repulsion of a payload (as measured by distant inertial observers in the rest frame of the source or in the initial rest frame of the payload) is given by (Reference 15):

$$\gamma^2 > \frac{3}{2}\psi\left[1 - \frac{L^2}{GMr}\left(\frac{\psi}{3} - \frac{GM}{rc^2}\right)\right] \tag{6}$$

In this expression, $\gamma \equiv (1 - \beta^2)^{-1/2}$ is the standard relativistic Lorentz transformation factor which is a function of the normalized relativistic velocity parameter $\beta = v/c$, $\psi \equiv \psi(r) = 1 - (2GM/rc^2)$ is the g_{00} (or time-time) component of the static Schwarzschild spacetime metric[2] of a source (or central) body of mass M, L is the constant specific angular momentum of a ballistic payload of mass m, and r is the radial distance of the approaching/receding payload from M. One can solve the inequality in Equation (6) for β (or v) under the condition that a payload far from M, such that $r >> b$ (b is the periapsis distance of the payload from M) and $r >> GM/c^2$, and find that the payload will become gravitationally repelled by M whenever $\gamma^2 > 3/2$ or $\beta > 3^{1/2}$. In order to derive an exact solution, Felber considered the case for which $M >> m$ so that the energy and momentum delivered to the payload has a negligible back-reaction on the source body's motion. And he found that a strong gravitational field is not required for antigravity propulsion because a weak-field solution achieves the same results.

Felber discovered another interesting facet about this new relativistic antigravity effect. He found that there is also an antigravity field that repels bodies in the backward direction with a strength that is one-half the strength of the antigravity field in the forward direction. Thus a stationary body will repel a test body that is radially receding from it at any $v > v_{crit}$. To delineate the propulsion benefit from this technique, Felber determined that the maximum velocity ($v_{pmax\text{-}wf}$) that can be imparted to a payload initially at rest by the weak (gravitational) field of a larger source mass moving toward the payload at constant $v > v_{crit}$ is $v_{pmax\text{-}wf} << c[\beta - (3\beta)^{-1}]$. For the strong-field case, the maximum velocity ($v_{pmax\text{-}sf}$) that can be imparted to the payload (initially at rest) by the larger source mass moving toward the payload at any constant v is $v_{pmax\text{-}sf} = \beta c$. Felber's analysis includes examples where he uses black holes for the large source mass.

This form of antigravity propulsion is not too surprising because Misner et al. (Reference 16), Ohanian and Ruffini (Reference 17), and Ciufolini and Wheeler (Reference 18) report that general relativistic calculations show that the time-independent Kerr (spinning black hole) gravitational field exhibits an inertial frame dragging effect similar to gravitational repulsive forces in the direction of a moving mass at relativistic velocities. This and Felber's exact solution are among the genre of Lense-Thirring type effects that produce antigravity forces. It is interesting to note that even though general relativity theory admits the generation of antigravity forces at relativistic velocities (Reference 19), they have not been seen in laboratory experiments

[2] A spacetime metric (ds^2) is a Lorentz-invariant distance function between any two points in spacetime that is defined by $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, where $g_{\mu\nu}$ is the metric tensor which is a 4×4 matrix that encodes the geometry of spacetime and dx^μ is the infinitesimal coordinate separation between two points. The Greek indices ($\mu,\nu = 0...3$) denote spacetime coordinates, $x^0...x^3$, such that $x^1...x^3 \equiv$ space coordinates and $x^0 \equiv$ time coordinate. The Schwarzschild metric is: $ds^2 = -(1 - 2GM/c^2r)c^2dt^2 + (1 - 2GM/c^2r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$. The corresponding metric tensor is a diagonal matrix: $g_{\mu\nu} = \mathrm{diag}[-(1 - 2GM/c^2r), (1 - 2GM/c^2r)^{-1}, r^2, r^2\sin^2\theta]$. (r,θ,φ) are the usual spherical polar coordinates in 3-dimensional space.

because repulsive force terms are second and higher-order in the source mass velocity. To invent a relativistic driver for a captured astronomical body in order to use it to launch payloads into relativistic motion presents a large technical challenge for future experimenters. For this reason, this paper will not consider this concept any further. However, it does serve the useful purpose of illustrating the unusual antigravity forces that can appear in Einstein's general relativity theory.

NEGATIVE ENERGY-INDUCED ANTIGRAVITY

Negative energy density and negative pressure are acceptable results both mathematically and physically in general relativity and quantum field theories, and negative energy/pressure manifests as gravitational repulsion (that is, antigravity). Negative energy is also known as a form of "exotic matter."

In classical physics the energy density of all observed forms of matter (fields) is non-negative. What is exotic about negative energy is that it must have negative energy density and/or negative flux (Reference 20). The energy density is "negative" in the sense that a given (exotic) matter field must have an energy density, ρ_E (= ρc^2, where ρ is the rest-mass density), that is less than or equal to its pressures/tensions, p_i (Reference 21,22).[3] In many cases, these equations of state are also known to possess an energy density that is algebraically negative; that is, the energy density and flux are less than zero. It is on the basis of these conditions that this material property is called "exotic." The condition for ordinary, classical (non-exotic) forms of matter that one is familiar with in nature is that $\rho_E > p_i$ and/or $\rho_E \geq 0$. These conditions represent two examples of what are variously called the "standard" energy conditions: Weak Energy Condition (WEC: $\rho_E \geq 0$, $\rho_E + p_i \geq 0$), Null Energy Condition (NEC: $\rho_E + p_i \geq 0$), Dominant Energy Condition (DEC), and Strong Energy Condition (SEC). These energy conditions forbid negative energy density between material objects to occur in nature, but they are mere hypotheses. Hawking and Ellis (Reference 23) formulated the energy conditions in order to establish a series of mathematical hypotheses governing the behavior of collapsed-matter singularities in their study of cosmology and black hole physics. More specifically, classical general relativity allows one to prove lots of general theorems about the behavior of matter in gravitational fields.

The bad news is that real physical matter is not "reasonable" because the energy conditions are in general violated by semiclassical quantum effects (occurring at order η) (Reference 22).[4] More specifically, quantum effects generically violate the average NEC (ANEC). Furthermore, it was discovered in 1965 that quantum field theory has the remarkable property of allowing states of matter containing local regions of negative energy density or negative fluxes (Reference 24). This violates the WEC, which postulates that the local energy density is non-negative for all observers. "Negative energy" has the unfortunate reputation of alarming physicists. This is unfounded since all the energy condition hypotheses have been experimentally tested in the laboratory and experimentally shown to be false – 25 years before their formulation (Reference 25).

[3] Latin indices (e.g., i, j, k = 1…3) that are affixed to physical quantities denote the usual 3-dimensional space coordinates, $x^1 \ldots x^3$, indicating the spatial components of vector or tensor quantities.
[4] Planck's reduced constant, $\eta = 1.055 \times 10^{-34}$ J·s.

Further investigation into this technical issue showed that violations of the energy conditions are widespread for all forms of both "reasonable" classical and quantum matter (Reference 26-30). Furthermore, Visser (Reference 22) showed that all (generic) spacetime geometries violate all the energy conditions. So the condition that $\rho_E > p_i$ and/or $\rho_E \geq 0$ must be obeyed by all forms of matter in nature is spurious. Violating the energy conditions commits no offense against nature. Negative energy has been produced in the laboratory and this will be discussed in the following sections.

Examples of Exotic or "Negative" Energy Found in Nature

The exotic (energy condition-violating) fields that are known to occur in nature are:

- Static, radially-dependent electric or magnetic fields. These are borderline exotic, if their tension were infinitesimally larger, for a given energy density (Reference 23,31).
- Squeezed quantum vacuum states: electromagnetic and other (non-Maxwellian) quantum fields (Reference 21,32).
- Gravitationally squeezed vacuum electromagnetic (or other field) zero-point fluctuations (Reference 33).
- Casimir effect; that is, the Casimir vacuum in flat, curved, and topological spaces (Reference 34-40).
- Other quantum fields/states/effects. In general, the local energy density in quantum field theory can be negative due to quantum coherence effects (Reference 24). Other examples that have been studied are Dirac field states: the superposition of two single particle electron states and the superposition of two multi-electron-positron states (Reference 41,42). In the former (latter), the energy densities can be negative when two single (multi-) particle states have the same number of electrons (electrons and positrons) or when one state has one more electron (electron-positron pair) than the other.

Cosmological inflation (Reference 22), cosmological particle production (Reference 22), classical scalar fields (Reference 22), the conformal anomaly (Reference 22), and gravitational vacuum polarization (Reference 26-29) are among many other examples that also violate the energy conditions. Since the laws of quantum field theory place no strong restrictions on negative energies and fluxes, then it might be possible to produce exotic phenomena such as faster-than-light travel (Reference 43-45), traversable wormholes (Reference 21,22,46), violations of the second law of thermodynamics (Reference 47,48), and time machines (Reference 22,46,49). There are several other exotic phenomena made possible by the effects of negative energy, but they lie outside the scope of this report. See Appendix A for more technical details on items 1 through 4.

Toy Model Estimate for Negative Energy-Induced Antigravity

For the purpose of this report, the discussion will be confined to how negative energy can be used to produce antigravity for the simplest case of counteracting the Earth's gravitational field. To counteract or otherwise reduce gravity merely requires the deployment of a thin spherical shell (bubble) of negative energy around an aerospace vehicle. This particular case study will serve as a useful illustrative comparison with the Newtonian antigravity case discussed in Section II-A.

Interest is only in the slow (non-relativistic) motion, weak (gravity) field regime that characterizes the physics of the Earth, Sun, other forms of solar system matter, most interstellar matter (excluding compact dense stars and black holes), and small test masses. In this case the time-time component of the Ricci curvature tensor ($R_{\mu\nu}$) is given by $R_{00} \approx G\rho/c^2 \approx (7.41 \times 10^{28})\rho\ m^{2}$. This is the primary quantity inside the general relativistic field equation[5] that encodes and measures the curvature of spacetime around a source of matter and characterizes the weak or strong gravity field regime for all forms of astronomical mass density (ρ). For example, the Earth's mass density is 5,500 kg/m^3 so $R_{00} \approx 4.08 \times 10^{-24}$ m^{-2}, which indicates that an extremely flat space surrounds the Earth and thus the system is within the weak field regime. Gravitational physics in the weak field regime is completely described by the standard Schwarzschild spacetime metric, which leads to the usual Newtonian and post-Newtonian gravitational physics.

Two simple approaches can be used to determine the negative energy density required to counteract the Earth's gravitational field: a) integrate the Einstein general relativistic field equation, or b) use an already derived result from general relativity that gives the repulsive force acceleration in terms of the spacetime metric components. For the first case, the generalized gravitational Poisson equation from the Einstein field equation is:

$$\begin{aligned} -R_{00}\sqrt{-g_{00}} &= \frac{4\pi G}{c^4} Tr\left(T_{\mu\nu}\right)\sqrt{-g_{00}} \\ &= \frac{4\pi G}{c^4} T^{\mu}_{\mu}\sqrt{-g_{00}} \\ &= \frac{4\pi G}{c^4}\rho_E^*, \end{aligned} \tag{7}$$

where the definition

$$\rho_E^* \equiv T^{\mu}_{\mu}\sqrt{-g_{00}}$$

is used, ρ_E^* ≡ rest-energy density + compressional potential energy (a.k.a. pressure), $g_{00} \equiv g_{00}(r)$ is the time-time component of the metric tensor $g_{\mu\nu}$, and $Tr(T_{\mu\nu}) \equiv T^{\mu}_{\mu}$ is the trace (sum of diagonal matrix elements) of the stress-energy-momentum tensor $T_{\mu\nu}$ (a matrix quantity that encodes the density and flux of a matter source's energy and momentum). Using tensor identities and grinding the algebra, Equation (7) can be re-written as

$$\nabla^2\sqrt{-g_{00}} = \frac{4\pi G}{c^4}\rho_E^* \tag{8}$$

[5] The Einstein field equation is: $G_{\mu\nu} \equiv R_{\mu\nu} - (1/2)g_{\mu\nu}R = -(8\pi G/c^4)T_{\mu\nu}$, where $G_{\mu\nu}$ is the Einstein curvature tensor and $R = R^{\mu}_{\mu}$ (the matrix trace of $R_{\mu\nu}$) is the Ricci scalar curvature. In simplest terms, this relation states that gravity is a manifestation of the spacetime curvature ($G_{\mu\nu}$) induced by a source of matter ($T_{\mu\nu}$).

where ∇^2 is the standard Laplace differential operator. The left-hand-side of Equation (8) is the gravitational potential. Integrating Equation (8) once over a region of space exterior to a ball (or thin spherical shell) of rest-energy density to obtain

$$\left|\nabla\sqrt{-g_{00}(r)}\right| = \frac{GM}{r^2} \equiv g \quad (acceleration,\ \mathrm{m/s^2}) \tag{9}$$

where the standard spherically symmetric spacetime (or Schwarzschild) coordinate system (t, r, θ, φ) in which time t, radial space coordinate r, and angular space coordinates (θ, φ) have their usual meaning is used.

The second approach (case b) can be derived by recalling that in the exterior Schwarzschild spacetime around a central mass M (a ball or thin spherical shell) is

$$\sqrt{-g_{00}(r)} = 1 - \frac{GM}{r} \tag{10}$$

Since the definition is given that $g \equiv \left|\nabla\sqrt{-g_{00}(r)}\right|$, then perform the radial derivative of Equation (10) and again arrive at Equation (9).

Since from special relativity $M = E/c^2$ (for a given rest-energy E), a negative energy state is identical to a negative mass state (Reference 50). Thus the mass M in Equation (9) can be replaced with the negative energy density $-\rho_E^* = -\rho c^2 = -Mc^2/V$ by using the volume ($V = 4\pi r^2 \delta r$) of a thin spherical shell of radius r and thickness δr, and rearrange quantities to solve for ρ_E^* to get the final result:

$$\begin{aligned}\rho_E^* &= \frac{-gc^2}{4\pi G\,\delta r} \\ &= \frac{-\left(1.05\times 10^{27}\right)}{\delta r} \quad \left(\mathrm{J/m^3}\right),\end{aligned} \tag{11}$$

where g is now the acceleration due to gravity near the Earth's surface. If one desires to use other geometries (for example, torus, cylinder, prism, cone, and pyramid) instead of a thin spherical shell, then Equation (11) will admit minor numerical adjustments to accommodate the relevant geometrical factors associated with different geometrical volumes. Equation (11) gives the negative energy density required to generate a repulsive gravitational force that counteracts the Earth's gravity field from the surface all the way up to LEO (since g in LEO is only a few percent smaller than on the surface). Any realistic value that one chooses for the bubble wall thickness δr will give a negative energy density that will always be on the order of the equivalent negative energy density of a dwarf star or neutron star. The technical challenge to implement this kind of antigravity, however, is daunting.

In the next section the case of a cosmological antigravity that is generated by a form of matter having a positive energy density and negative pressure is discussed.

COSMOLOGICAL ANTIGRAVITY

It turns out that there is already a naturally occurring antigravity force that acts throughout the universe. Actually, this force acts upon the entire spacetime structure of the universe, and it is called cosmological inflation. Cosmological inflation causes the universe to expand at an ever accelerating rate. In what follows, the nature of this cosmological antigravity force and its potential aerospace propulsion application is examined.

Pressure as a Source of Gravity

Newtonian gravitation is modified in the case of a relativistic perfect-fluid (where $p \ll \rho_E$ cannot be assumed). The stress-energy tensor $T^{\mu\nu}$ for this case is (Reference 16):

$$T^{\mu\nu} = (\rho_E + p)U^{\mu}U^{\nu} - pg^{\mu\nu} \tag{12}$$

where ρ is the fluid mass density, $\rho_E \equiv \rho c^2$ is the fluid rest-energy density (or just energy density), p is the fluid pressure, U^{μ} is the 4-velocity vector of the fluid, and $g^{\mu\nu}$ is the metric tensor. The Einstein general relativistic field equation using the identity $g^{\mu}{}_{\mu}$ = 4 to obtain $R = (8\pi G/c^4)T$, which is the Ricci curvature scalar can be contracted. And so Equation (12) becomes $T = \rho_E - 3p$, which is just the trace of $T^{\mu\nu}$. Since $T = \rho_E - 3p$, a modified Newtonian gravitational Poisson equation is produced:

$$\nabla^2\phi = 4\pi G(\rho_E + 3p) \tag{13}$$

where ϕ is the gravitational potential. It should be noted that the energy density and pressure are kept as separate terms as opposed to Equations (7) and (8) in the previous section. Equation (13) means that a gas of particles all moving at the same speed u has an effective gravitational mass density of $\rho(1 + u^2/c^2)$. Thus, for example, a radiation-dominated fluid generates a gravitational attraction twice as strong as one predicted by Newtonian gravity theory according to Equation (13).

Vacuum Energy of Einstein's Cosmological Constant

A major consequence of the Einstein field equation is that pressure p becomes a source of gravitational effects on an equal footing with the energy density ρ_E. One consequence of the gravitational effects of pressure is that a negative-pressure equation of state that achieves $\rho_E + 3p < 0$ in Equation (13) will produce gravitational repulsion (that is, antigravity). The Einstein field equation that includes a cosmological constant Λ is:

$$G^{\mu\nu} + \Lambda g^{\mu\nu} = -\frac{8\pi G}{c^4}T^{\mu\nu} \tag{14}$$

where $G^{\mu\nu}$ is the Einstein curvature tensor. The Λ term, as it appears in Equation (14), represents the curvature of empty space. Now if one moves this term over to the right-hand-side of Equation (14), which has become widespread practice in modern times, then

$$G^{\mu\nu} = -\left(\frac{8\pi G}{c^4}T^{\mu\nu} + \Lambda g^{\mu\nu}\right) \tag{15}$$

whereby this term now behaves like the stress-energy tensor of the vacuum, $T^{\mu\nu}_{\text{vac}}$, which acts as a gravitational source:

$$T^{\mu\nu}_{\text{vac}} = \frac{\Lambda c^4}{8\pi G} g^{\mu\nu} \tag{16}$$

One should note that the absence of a preferred frame in special relativity means that $T^{\mu\nu}_{\text{vac}}$ must be the same (that is, isotropic or invariant) for all observers. There is only one isotropic tensor of rank 2 that meets this requirement: $\eta^{\mu\nu}$ (the Minkowski flat spacetime metric tensor in locally inertial frames). So in order for $T^{\mu\nu}_{\text{vac}}$ to remain invariant under Lorentz transformations, the only requirement is that it must be proportional to $\eta^{\mu\nu}$. But this generalizes in a straightforward way from inertial coordinates to arbitrary coordinates by replacing $\eta^{\mu\nu}$ with $g^{\mu\nu}$, thus justifying the curved spacetime metric tensor in Equation (16). By comparing Equation (16) with the perfect-fluid stress-energy tensor in Equation (12), one finds that the vacuum looks like a perfect fluid with an isotropic pressure p_{vac} opposite in sign to the energy density ρ_{vac}. Therefore, the vacuum must possess a negative-pressure equation of state (according to the first law of thermodynamics):

$$p_{\text{vac}} = -\rho_{\text{vac}} \tag{17}$$

The vacuum energy density should be constant throughout spacetime, since a gradient would not be Lorentz invariant. So by substituting Equation (17) into $\rho_E + 3p$, the following is produced

$$\begin{aligned}\rho_{\text{vac}} + 3p_{\text{vac}} &= \rho_{\text{vac}} + 3\left(-\rho_{\text{vac}}\right) \\ &= -2\rho_{\text{vac}} \\ &< 0.\end{aligned} \tag{18}$$

The vacuum equation of state is therefore manifestly negative. Last, when incorporating ρ_{vac} into the Einstein field equation as a gravitational source term, and comparing its corresponding (Lorentz invariant) stress-energy tensor $\rho_{\text{vac}}g^{\mu\nu}$ with Equation (16), then the usual identification (or definition) is made that:

$$\rho_{\text{vac}} \equiv \frac{\Lambda c^4}{8\pi G} \tag{19}$$

Thus the terms "cosmological constant" and "vacuum energy" are essentially interchangeable in this perspective and mean the same thing (whereupon $\rho_{\text{vac}} \equiv \rho_\Lambda$), which is seen in the present-day cosmological literature.

By substituting Equation (19) into Equation (17), one observes that a positive Λ will act to cause a large-scale repulsion of space (because this gives a negative vacuum pressure), whereas a negative Λ (giving a positive vacuum pressure) will cause a large-scale contraction of space. Because Λ is a constant, the vacuum energy is a constant (that is, time independent). This then implies a problem with energy conservation in an expanding universe since one expects that energy density decreases as a given volume of space increases, which is the case for the ordinary matter and cosmic microwave background that is observed in extragalactic space. In other words, the matter and radiation energy densities decay away as the universe expands while the vacuum energy density remains constant.

The cure for this apparent energy conservation problem is the vacuum equation of state given by Equation (17). A negative pressure is something like a tension in a rubber band. It takes work to expand the volume rather than work to compress it. The proof of this is as follows (Reference 51): the energy created in the vacuum by increasing (expanding) space by a volume element dV is $\rho_{vac}dV$, which must be supplied by the work done by the vacuum pressure $-p_{vac}dV$ during the expansion of space, therefore $p_{vac} = -\rho_{vac}$. In other words, the work done by the vacuum pressure maintains the constant vacuum energy density as space expands. Therefore, the vacuum acts as a reservoir of unlimited energy that provides as much energy as needed to inflate any region of space to any given size at constant energy density.

Dark Energy

Dark energy is an easily misunderstood form of energy in cosmology. There are two sets of evidence pointing toward the existence of something else beyond the radiation and (ordinary and dark) matter itemized in the overall cosmic energy budget.[6] The first comes from a simple budgetary shortfall. The total energy density of the universe is very close to critical. This is expected theoretically and it is observed in the anisotropy pattern of the cosmic microwave background (CMB). Yet, the total matter density inferred from observations is 26 percent of critical.[7] The remaining 74 percent of the energy density in the universe must be in some smooth, unclustered form that is dubbed "dark energy." The second set of evidence is more direct. Given the energy composition of the universe, one can compute a theoretical distance vs. redshift diagram. This relation can then be tested observationally.

Riess et al. (Reference 52) and Perlmutter et al. (Reference 53) reported direct evidence for dark energy from their supernovae observations. Their evidence is based on the difference between the luminosity distance in a universe dominated by dark matter and one dominated by dark energy. They showed that the luminosity distance is larger for objects at high redshifts in a dark energy-dominated universe. Therefore, objects of fixed intrinsic brightness will appear fainter if the universe is composed of dark energy. The two groups measured the apparent magnitudes of a few dozen Type Ia supernovae at redshifts $z \leq 0.9$, which are known to be standard distance candles (meaning they have nearly identical absolute magnitudes at any cosmological redshift-

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[6] Dark matter and dark energy are not to be confused. Dark matter is a non-luminous, non-absorbing, non-baryonic form of matter that only interacts with all other forms of matter via gravitational and weak nuclear forces. Dark matter has a positive rest-energy density and a nearly negligible positive pressure. Thus, it has no beneficial application for breakthrough propulsion physics.

[7] 26% total matter density = 4% ordinary (baryonic) matter + 22% dark matter.

distance).[8] The supernovae data strongly disfavored (with high confidence) the flat matter-dominated ($\Omega_m = 1$, $\Omega_\Lambda = 0$) universe and the pure open universe ($\Omega_m = 0.3$, $\Omega_\Lambda = 0$) models.[9] After this discovery, a lot of attention was paid to choosing an appropriate name for this new energy. "Quintessence" was one good choice because it expresses the fact that, after cosmological photons, baryons, neutrinos, and dark matter, there is a fifth essence in the universe. More recently, "dark energy" is used more often, with quintessence referring to the subset of models in which the energy density can be associated with a time-dependent scalar field or a time-dependent cosmological vacuum energy.

In analyzing the cosmological modeling results suggested by the Type Ia supernovae data, it becomes apparent that the only form of dark energy budgeted for in the models is the cosmological constant. To consider other possibilities one evaluates the time evolution of the general relativistic conservation law for energy, $\nabla_\mu T^\mu_\nu = \nabla_\mu T^\mu_0 = 0$, where $\nu = 0$ to signify time evolution and ∇_μ is the covariant derivative (or spacetime curvature gradient), in an expanding universe as applied to the cosmological constant (Reference 16):

$$\frac{\partial \rho_E}{\partial t} + \frac{\dot{a}}{a}\left[3\rho_E + 3p\right] = 0 \tag{20}$$

where a is the scale factor of the universe and $\dot{a}$ is the time derivative of a. Equation (20) is derived using Equation (12) in the case of a perfect isotropic fluid where there is no gravity and velocities are negligible such that $U^\mu = (1, 0, 0, 0)$, and the energy density and pressure evolve according to the continuity and Euler equations. The only way Equation (20) can be satisfied with constant energy density is if the pressure is defined by Equation (17). One might imagine energy with a slightly different pressure and therefore energy evolution. Define the equation of state w:

$$w = \frac{p}{\rho_E} \tag{21}$$

A cosmological constant corresponds to $w_\Lambda \equiv w_{vac} = -1$, matter (ordinary and dark) to $w_{matter} \approx 0$, and radiation to $w_{rad} = 1/3$.[10] The earlier Riess and Perlmutter supernovae data (fixing the universe to be flat) showed that values of $w_{de} > -0.52$ for dark energy are strongly disfavored. In fact, Riess and a team of collaborators (a.k.a. the "Higher-*Z* team") recently published new observational data and analysis that includes a much larger survey of Type Ia supernovae that are at much higher cosmological redshift (Reference 54). The measured spectra of ancient ($z \geq 1$, or up to **10** billion light-years distance or a look-back time of up to **10** billion years ago) and recent ($z \leq 0.1$, or $\leq$ **1** billion light-years distance or a look-back time of $\leq$ **1** billion years ago) were compared and showed that there was no evolutionary change in the physics that drives Type Ia supernovae explosions and their subsequent spectral luminosity output. This establishes

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[8] In cosmology, the redshift z serves as a surrogate for distance (in light-years) or look-back time.

[9] Ω_m = ratio of energy density contained in matter (as measured today) to the critical energy density; $\Omega_\Lambda = \Omega_{vac}$ = ratio of energy density in a cosmological constant to the critical energy density; $\rho_{cr} \equiv 3H_0^2/8\pi G$ is the critical energy density, where H_0 is the present-day Hubble expansion rate.

[10] Non-relativistic (ordinary and dark) matter has a very tiny positive pressure, $p \propto T_{emp}/m$ (T_{emp} is absolute temperature, m is mass), while a relativistic gas (of radiation) has $p = \rho_E/3 > 0$.

the efficacy of using Type Ia supernovae as a standard distance candle for cosmological dark energy surveys. The Higher-*Z* team's results also concluded, with 98 percent confidence, that $w_{de} = -1.0$, and that this is a perpetual constant (over at least 10 billion years time) (Reference 54). This result falsifies all quintessence models for cosmology. Therefore, a cosmological constant is consistent with the dark energy data to a high degree of precision and statistical confidence whereby one can now state that dark energy is the vacuum energy of Einstein's cosmological constant because $w_{de} = w_{\Lambda} = -1$ (Reference 55,56).

Equation (20) can be integrated to find the evolution of the dark energy density, $\rho_{de} = \rho_{\Lambda}$, as a function of the cosmological scale factor a:

$$\rho_{de} \propto \exp\left\{-3\int_{a} \frac{da'}{a'}\left[1 + w_{de}(a')\right]\right\} \qquad (22)$$

where a' is the dummy integration variable for the scale factor. Since $w_{de} = -1$ ($= w_{\Lambda}$) is a constant in Equation (22), then $\rho_{de} \propto a\exp[-3(1+w_{de})]$ or $\rho_{de} = \rho_{\Lambda} \propto a^{0}$. This is exactly what is expected on the basis of previous analysis in Section III-D-2. For a comparison with this result, one should note that $\rho c^2 \propto a^{-3}$ for (ordinary and dark) matter and $\rho_{rad} \propto a^{-4}$ for radiation such that $\rho c^2 \to 0$ and $\rho_{rad} \to 0$ as $a \to \infty$ while $\rho_{de} = \rho_{\Lambda}$ remains constant.

Antigravity Propulsion Application of Dark/Vacuum Energy

If one could somehow harness a local amount of dark/vacuum energy, then use can be made of its negative pressure property to produce an antigravity propulsion effect? To answer this question one can use the estimated value for $\rho_{de} = \rho_{\Lambda} \approx 2.4\rho_0 c^2 \approx 10^{-9}$ J/m^3, where ρ_0 is the present-day value of the total cosmological mass density of (ordinary and dark) matter (Reference 54,57). Using this number one can work through the math and estimate that the total amount of dark/vacuum energy contained within our solar system amounts to the mass equivalent of a small asteroid. This means that its repulsive gravitational influence upon planetary orbital dynamics inside the solar system is completely inconsequential. Only on the extragalactic-to-cosmological scale will its repulsive gravitational property achieve strong enough influence over matter and spacetime. On this basis, one can conclude that it is highly unlikely, if not impossible, that one will be able to invent a technology in the near future that can acquire and exploit a near-cosmological amount of dark/vacuum energy to implement a useful antigravity propulsion system.

IV. Quantum Antigravity Propulsion Concepts

Quantum antigravity can be found within the very large genre of quantum gravity theories in which repulsive gravity terms appear as quantum corrections to the classical Newtonian gravitational force law. Generally, one can derive such correction terms by quantizing the Einstein general relativistic field equation or by starting with a particular type of quantum field theory (for example, supersymmetric field theory, quantized 5-dimensional Kaluza-Klein unified field theories, quantum superstrings/D-Brane theory, quantum loops or knots, and Yang-Mills theories) and work backwards to find the corresponding gravity theory. The particular mathematical form and quantitative magnitude that quantum correction terms can have totally depends upon the quantization procedure and order of approximation used in a given quantum gravity theory. However, the linearized semi-classical quantum gravity theory is related to Einstein's classical nonlinear General Relativity Theory whereby the former uniquely implies the latter provided that the graviton, which exchanges the gravitational force between two massive particles or photons, is a pure spin-2 particle. In this theory, the stress-energy tensor of the source matter fields is quantized while gravitation (via the Einstein curvature tensor) is still treated classically. Semi-classical quantum gravity is a quantum field theory in curved spacetime that has been successful in reproducing a few of the predictions and many of the foundational precepts of General Relativity Theory.

A particular example of what a quantum antigravity correction term looks like was derived in 1984 by R. L. Forward and the author, with instruction provided by R. P. Feynman and M. Scadron, during a summer quantum gravity seminar sponsored by the Hughes Research Labs in Malibu, CA. One began by studying the Feynman quantization procedure for the case of single-photon exchange between two charged particles, which tells us about the underlying nature and quantum corrections to the static Coulomb force. From this study discovered that the same is also true for the case of single-graviton exchange between two massive spin-0 particles in connection with the static Newtonian force. By applying Feynman's quantization procedure (Reference 58-60) to the linearized Einstein field equation in the nonrelativistic limit, the following static graviton-exchange potential, $V_{\text{grav}}(r)$, for two spin-0 particles undergoing a gravitational interaction can be derived:

$$V_{\text{grav}}(r) = -\frac{G m_1 m_2}{r} + \frac{4\pi G \hbar^2}{c^2}\delta^3(\boldsymbol{r}) \tag{23}$$

where m_1 and m_2 are the masses of the interacting particles, r is their radial separation, and $\delta^3(\boldsymbol{r})$ is the 3-dimensional Dirac δ-function with $\boldsymbol{r}$ the position vector of some reference point in space. The first term in Equation (23) is immediately recognized as the attractive Newtonian gravitational potential while the second quantum correction term is repulsive. Also, the second term is independent of the interacting particle masses and can only be measured for bound quantum *s*-states because the product of the coefficient $4\pi(G\hbar^2/c^2) \approx 10^{\,94}$ with the δ-function gives only a minute physical effect at the atomic scale. The second term happens to be analogous to the usual quantum correction to the Coulomb or nuclear force. If the two particles were to have non-zero quantum spin, then $V_{\text{grav}}(r)$ will be modified by additional spin-orbit and spin-spin correction terms. Furthermore, there are additional velocity-dependent corrections to $V_{\text{grav}}(r)$ that generate the general relativistic post-Newtonian modifications of the classical equation of motion of a particle in a gravitational field.

But the most important characteristic to observe about the quantum antigravity correction term in Equation (23) is that its magnitude is incredibly minute, only affecting bound quantum *s*-states. In general, quantum gravity correction terms at any level of approximation, whether gravitationally repulsive or attractive, will have coefficients ~ $G(\hbar^\delta/c^\kappa)$ (for $\delta, \kappa > 1$), and therefore will not have a measurable impact on any macroscopic system that embodies any form of propulsion. Because these quantum corrections are so minute, and because there is no single universally accepted quantum gravity theory to work with, investigators have had little reason to look into the potential application of quantum gravity correction terms to antigravity propulsion physics.

However, this isn't the entire story because there are many interesting quantum field theoretic phenomenon that exist outside of that which arises in quantum corrections to Newtonian gravity. In what follows, the recent discovery of antigravity forces that arise within both QED vacuum fluctuation and nonretarded quantum interatomic dispersion force theories in curved spacetime are reviewed.

ANTIGRAVITY VIA QUANTUM VACUUM ZERO-POINT FLUCTUATION FORCE

Calloni et al. (Reference 61,62) explored the possibility of verifying the equivalence principle for the zero-point energy of quantum electrodynamics (QED). They used semi-classical quantum gravity theory to evaluate the net force produced by quantum vacuum zero-point fluctuations (ZPF) acting on a rigid Casimir cavity in a weak gravitational field. Their analysis assumed the rigid Casimir cavity to be a non-isolated system at rest in the Earth's gravitational field, which is modeled using the standard Schwarzschild spacetime metric geometry, so that they could evaluate the regularized (or renormalized) stress-energy tensor, $\left\langle T^{\mu\nu}_{\text{vac}} \right\rangle_{\text{ren}}$,[11] of the quantized vacuum electromagnetic field between two plane-parallel ideal metallic plates lying in a horizontal plane. $\left\langle T^{\mu\nu}_{\text{vac}} \right\rangle_{\text{ren}}$ encodes the Casimir Effect which has a negative energy density and a negative pressure along the vertical (acceleration) axis between the plates. (See Appendix A for more information about the Casimir Effect.) Their results agreed with the equivalence principle because they showed that quantum vacuum ZPF (that is, virtual quanta) do gravitate because the energy of each ZPF mode is redshifted by the factor $(-g_{00})^{1/2} = \left[1 - \left(2GM/c^2 r\right)\right]^{1/2}$ even though the modes remain unchanged. In other words, the electromagnetic vacuum state in a weak gravitational field is redshifted. This effect remains true for strong gravitational fields.

The resulting antigravity force (F_{CasGrav}) derived by Calloni et al. is (Reference 62):

$$
\begin{aligned}
F_{\text{CasGrav}} &= \frac{\pi^2 A \hbar g}{180 c d^3} \\
&\approx \left(1.89 \times 10^{43}\right) \frac{A}{d^3}
\end{aligned}
\tag{24}
$$

in Newtons (N), where A is the area of the plates and d is their separation. Equation (24) states that a Casimir device in a weak gravitational field will experience a tiny push in the upwards direction (that is, the opposite direction with respect to the Earth's gravitational acceleration). This is consistent with the interpretation that the negative Casimir energy in a gravitational field behaves like a negative mass (Reference 63). F_{CasGrav} is actually the sum of two separate force terms: the first term arises from the Casimir energy encoded in $\left\langle T^{00}_{\text{vac}} \right\rangle_{\text{ren}}$ which is interpreted as the Newtonian repulsive force on an object with negative energy, and the second term arises from the pressure along the vertical (acceleration) axis which is interpreted as the mass contribution of the spatial part of the stress-energy tensor. To evaluate F_{CasGrav} for the case of any gravitating body of interest, one must replace *g* in Equation (24) with Equation (2).

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[11] The angular brackets denote the quantum (vacuum state) expectation value of the stress-energy tensor $T^{\mu\nu}$. Also note that stress-energy is synonymous with energy-momentum.

Calloni et al. further point out that a real Casimir cavity is an isolated system in which the actual (total) resulting force is the Newtonian force on the sum of the rest-Casimir energy and rest-mechanical mass whereby the contribution of the vacuum ZPF leads to a gravitational repulsion (F_{CGexp}) on the Casimir device that is given by (Reference 62):

$$\begin{aligned} F_{\mathrm{CGexp}} &= \frac{1}{4} F_{\mathrm{CasGrav}} \\ &\approx \left(4.73\times10^{-44}\right)\frac{A}{d^3} \quad (\mathrm{N}), \end{aligned} \tag{25}$$

which is the force that should be experimentally tested. Equation (25) takes into consideration that the contribution to the total force on a real cavity resulting from the spatial part of the stress-energy tensor is balanced by the contribution from the mechanical stress-energy tensor. Given that the typical dimensions of a Casimir device are very small, it appears that F_{CGexp} will be very difficult, if not impossible, to measure using present-day lab technology.

However, Calloni et al. propose an experimental device that could significantly magnify the repulsive force up to a measurable scale. Their proposed device is a multilayered series of rigid Casimir cavities with each cavity consisting of two thin metallic disks that are separated by a dielectric material which is inserted to maintain rigidity. They suggest SiO_2 for the dielectric material because it is an efficient dielectric with low absorption over a wide range of frequencies, and it is an inexpensive material that is easy to fabricate into layers. The introduction of the dielectric material is equivalent to enlarging the optical path length by the refractive index n so that the cavity plate separation $d \propto nd$. The Casimir Effect has been tested down to plate separations ~ 60 nm while separations ≤ 10 nm is possible with present technology. But at ≤ 10 nm distances dielectric absorption and finite conductivity are expected to decrease the effective Casimir pressure compared to a cavity comprised of perfect mirrors. For example, a plate separation of 6.5 nm corresponds to a decreasing factor (ζ) of 0.07 for plates made of aluminum. Finite temperature and plate surface roughness could also introduce additional corrections to the Casimir pressure. Calloni et al. propose to magnify the total force by using $N_l = 10^6$ layers of rigid cavities with each cavity having a diameter of 35 cm and thickness of 100 nm, for a total device thickness of 10 cm.

All these engineering factors taken together led Calloni et al. to recast F_{CGexp} into the following new form (Reference 62):

$$\begin{aligned} F_{\mathrm{CGexp}} &\approx \zeta N_l \frac{\pi^2 A \hbar g}{720 c (nd)^3} \\ &\approx \left(4.73\times10^{-44}\right)\frac{\zeta N_l A}{(nd)^3} \end{aligned} \tag{26}$$

in Newtons. Calloni et al. also suggest that a feasible experiment will require modulating F_{CGexp} in order to obtain a measurable force. They are investigating the possibility of modulating ζ by varying the temperature in order to induce a periodic transition from conducting state to superconducting state. They estimate that doing this could achieve $\zeta_{max} \approx 0.5$, and thus produce a force $F_{\mathrm{CGexp}} \sim 10^{\ 14}$ N at a modulation frequency on the order of tens of mHz for d = 5 nm and n = 1.46 (for SiO_2 dielectric). This result is more than two orders of magnitude larger than the force which the VIRGO gravitational wave antenna is expected to detect at several tens of Hz. If one could fabricate a device consisting of 10^9 layers, then $F_{CGexp} \sim 10^{-11}$ N. This suggests that cavities made from thin-film deposited surfaces or photonic band-gap materials would be the best approach for fabricating a multilayer Casimir device.

Bimonte et al. (Reference 63-65) also derived Equation (24) for this very same problem by using Green-function techniques in the Schwinger-DeWitt quantum ether prescription for $\langle T^{\mu\nu}_{vac} \rangle_{ren}$ in a curved spacetime. They also computed the weak gravitational field-induced correction terms for the Casimir pressure on the plates, $\langle T^{\mu\nu}_{vac} \rangle_{ren}$, and the total energy ($E_{CasGrav}$) stored in the Casimir device which is given by (Reference 63,64):

$$E_{CasGrav} = -\frac{\pi^2 A \hbar c}{720 d^3}\left(1 + \frac{5}{2}\frac{g d}{c^2}\right) \tag{27}$$

in Joules (J). The correction terms for the different (measurable) physical quantities of interest are generally $\sim g/c^2$.

Finally, Calloni et al. point out that the overriding concern with performing an experiment to test F_{CGexp} is whether cavities can be made sufficiently rigid, if the effect of surface roughness and defects can be quantified to improve the force estimate, and if the necessary signal modulation can be achieved in the lab. However, micro- and nano-manufacturing is maturing to the point where rigidity, surface roughness, and close plate separations are becoming routinely controllable. While the numerical estimate for F_{CGexp} is quite feeble, it is still significant since it is at the very low end of the macroscopic scale, and it might be possible to devise advanced methods to magnify the force to a magnitude that benefits a propulsion application. However, the upward force will have to be larger than the weight of the propulsion system in order to achieve levitation. This could be very difficult to do, but this is a concept that is ripe for further exploration.

ANTIGRAVITY VIA NONRETARDED QUANTUM INTERATOMIC DISPERSION FORCE

Pinto (Reference 66) evaluated the net lifting force produced by nonretarded electrostatic dipole-dipole interactions (that is, nonretarded van der Waals dispersion forces) acting on a quantum system of polarizable particles in a curved spacetime. The foundation of Pinto's study was the original discovery made by Fermi (Reference 67) that classical electrostatic theory must be reformulated in a curved spacetime in order to properly evaluate the effects of gravitation upon the Coulomb electric field of a single charged particle. In this case, the Laplace equation of electrostatics for a single charged particle can be generalized in the presence of a gravitational field and then extended to show that a system of classical charged particles undergoes a gravity-induced self-lifting force. Fermi and other investigators arrived at this counterintuitive result by computing the gravity-induced self-force acting on an isolated electric dipole in a weak gravitational field and showing that the self-force (times dipole size) is exactly equal to the gravitational equivalent of the electrostatic internal energy of the dipole.

The net gravity-induced (electrostatic levitation) self-force (F_{DipGrav}) is given by (Reference 66):

$$F_{\text{DipGrav}} = \left(\frac{q_e^2}{4\pi\varepsilon_0 r}\right)\left(\frac{g}{c^2}\right) \quad \text{(N)} \tag{28}$$

where q_e is the electric charge on a particle and r is the radial distance between two charged particles in the dipole. There is an additional term of order g^2/c^4 in F_{DipGrav} that is neglected because it is negligible in magnitude. Equation (28) states that an electric dipole will experience a push in the upwards direction (opposite direction with respect to the Earth's gravitational acceleration); that is, the dipole undergoes self-acceleration in which one charged particle in the dipole appears to be chasing the other charged particle. As an example, for a dipole comprised of two charges (for example, an electron-proton system) held at fixed r to levitate in the Earth's gravitational field, r would have to be ~ 10^{15} m (the size of an atomic nucleus). An experiment to test this prediction on such a small scale is too difficult to control or measure.

An energy analysis done by Pinto showed that there is a distance *r* between two charges (each of rest-mass m_0) in a dipole (of mass $M_{\text{dip}} = 2m_0$) such that their electrostatic potential energy, $U_{\text{dip}} = -q_e^2/4\pi\varepsilon_0 r$, becomes equal to the unrenormalized mass of the system as $r \to \infty$. At this distance, the effective total gravitational mass $M_{\text{dip}} + U_{\text{dip}}/c^2 = 0$ and the self-force alone can support the dipole at rest against its own weight. The self-acceleration of the dipole is such that the acceleration process can continue indefinitely, which poses a problem for energy conservation because the dipole can be left to self-accelerate for an arbitrary period of time and then stopped to harness the resulting kinetic energy. This process could be used to extract unlimited energy from the system. Pinto claims that there is no conflict with energy conservation because the renormalized inertial mass of the accelerating system is $M_{\text{dip-ren}} = M_{\text{dip}} + U_{\text{dip}}/c^2 = 0$ and the total energy of the system is zero at all times regardless of speed. This claim requires reevaluation because there are subtle boundary conditions involved that might have been overlooked in the analysis.

Fermi's discovery led to a new subfield of research devoted to the study of electrodynamics and dipole and interatomic dispersion forces in a curved spacetime. Pinto's theoretical program extended the result of these studies by considering a system of polarizable atoms and adopting an approach in which the effect of a gravitational field in general relativity is modeled as an effective optical medium. In other words, the spacetime vacuum is treated as a non-uniform optical medium with a varying index of refraction that defines the components of a flat spacetime metric geometry (Reference 68). There is no spacetime curvature due to sources of matter in this model, instead its equivalent general relativistic effects (that is, gravitation) are produced by varying the vacuum index of refraction, comprised of the vacuum electromagnetic permittivity and permeability constants, in response to the presence of matter sources. Pinto's lengthy analysis gives the van der Waals dispersion self-force for two polarizable atoms in a curved spacetime (that is, a weak gravitational field) as (Reference 66):

$$
\begin{aligned}
F_{\mathrm{vdWGrav}} &= -U_{\mathrm{vdW}}^{(0)}\left(\frac{g}{c^2}\right) \\
&= -\left(-\frac{6\,q_e^2 a_0^5}{4\pi\varepsilon_0 r^6}\right)\left(\frac{g}{c^2}\right) \\
&\approx \left(2.44\times 10^{\ 57}\right)\frac{q_e^2}{r^6}
\end{aligned}
\qquad (29)
$$

in Newtons, where a_0 is the Bohr radius ($5.292 \times 10^{\ 11}$ m), r is the radial distance between two atoms, and $U_{\mathrm{vdW}}^{(0)}$ is the flat spacetime van der Waals (interatomic potential) interaction energy to second-order in quantum perturbation theory. Pinto used Equation (29) to estimate the gravity-induced self-acceleration (a_{lift}) for the case of two hydrogen atoms in their ground state at $r = 20a_0$, and found that $a_{\mathrm{lift,H}} = F_{\mathrm{vdWGrav}}/2m_{\mathrm{H}} \approx 4 \times 10^{-15}$ *m/s*^{2} (m_{H} = mass of hydrogen atom). For the case of two positronium (Ps) atoms, he found that $a_{\mathrm{lift,Ps}} \approx 8 \times 10^{\ 12}$ *m/s*^{2}.

Pinto's strategy is to dramatically magnify F_{vdWGrav} to a large enough magnitude that it becomes viable for propulsion applications. He claims that this can be done by manipulating $U_{\mathrm{vdW}}^{(0)}$, which depends on the atomic polarizability and is strongly affected by the quantum state in which the atoms are prepared. Interatomic forces can also be manipulated by means of external electromagnetic fields that can transform van der Waals forces into a first-order interaction. He evaluated a number of schemes and settled on the following techniques for manipulating dispersion forces: 1) excitation of polarizable atoms to Rydberg states in external time-dependent electric fields, 2) polarizability resonant enhancement by laser radiation, and 3) laser-induced near-zone orientational average of the dispersion force. Also, in order to generate a macroscopic self-lifting force, it will be necessary to apply these techniques to a cluster of trapped atoms because the total self-lifting force acting on the center-of-mass of a trapped gas composed of N_a identical polarizable atoms is N_a^2 times the self-lifting force acting on a single pair of interacting atomic dipoles. Item 1 has a two-part contribution to the magnification of the self-lifting force: 1) one part from $\alpha^2(\omega)E^2$ due to the effect of external time-dependent electric fields on atomic polarization, where $\alpha(\omega)$ is the atomic polarizability as a function of the electric field frequency ω and *E* is the electric field intensity; 2) another part from using highly-excited Rydberg atoms (with principal quantum number n_p >> 1 and Bohr radius $a_n = n_p^2 a_0$) whose polarizability scales as n_p^7. Item 2 leads to a magnification by factors of $\alpha(\omega)/\alpha_0 \approx 10^3$ - 10^5 (α_0 is the static value of the polarizability) via detuning of the (laser) excitation radiation frequency from the nearest atomic transition resonance of the atoms in the trapped cluster. Item 3 leads to a further magnification due to the effect of the incident laser radiation on the dispersion force being averaged over all directions, which changes the interatomic potential ($\propto 1/r^6$) into a gravity-like $1/r$ potential.

Pinto's study suggests that the combined effect of items 1 - 3 will magnify the self-lifting force to the point where a cluster of trapped atoms will not only hover unsupported in the Earth's gravitational field, but will also generate an additional upward thrust. On the basis of extensive theoretical and empirical studies, along with the typical parameters for laboratory laser and optical atomic matter trap technologies, he estimates that $a_{\mathrm{lift}} \geq 1.5$-$g$ (in the upward direction). Trapped atom gravimeters can be used to observe this effect in the lab. Pinto also points out that other polarizable systems such as nanoparticles, microspheres, and quantum dots can be used in place of atoms. The trapping of latex spheres into a form of optical matter by means of intense laser radiation has already been demonstrated in the lab. In addition, an analogy to the item 1 - 3 manipulations that produce dramatically enlarged polarizabilities in trapped interacting nanoparticles and microspheres have also been demonstrated in the lab.

Pinto proposes a levitation propulsion thruster in which the combined system of trapped interacting polarizable particles and external confining fields forms a single thruster element comprising a fraction of the mass of the entire vehicle. The reaction of the self-lifting force exerted by this element against the external confining fields results in the transfer of force (thrust) to the entire vehicle. In order to achieve levitation, this requires that the upward thrust per polarizable particle be larger than its own weight if the fraction of the thrusting mass is smaller than the mass of the rest of the vehicle. The propulsive levitation condition is expressed as (Reference 66): $F_{thrust} = (M_{veh} + m_A N)g$ or $F_{thrust}/m_A N g \geq 1$, where F_{thrust} is the total gravity-induced thrust, M_{veh} is the vehicle mass, m_A is the mass of individual polarizable particles, and N is the total number of trapped polarizable particles.

Pinto identified numerous technical challenges that will have to be overcome before this concept can be put to practice. One challenge is that polarizability resonant enhancement also leads to atomic transitions and decay which result in the recoil and evaporation of atoms from inside the trap. Another is the difficulty of maintaining continued confinement of a trapped cluster of polarizable particles in a specific 3-dimensional array while the cluster is simultaneously opposing the amplified interatomic forces and producing thrust. The confinement lifetime of trapped polarizable particles is finite and there is the possibility that these particles might be evaporated away or destroyed in a time that is too short to deliver the required thrust to the vehicle. Therefore, a scheme for active repopulation of the trapped cluster will have to be developed. The design of particle cluster traps and associated external confinement fields are of primary importance to determine the effective thrusting time of every polarizable particle. In addition, Rydberg atoms suffer from finite radiative lifetimes and are sensitive to external perturbations, so dispersion force manipulation might lead to the ionization of atoms. Tradeoffs will have to be made between all of the relevant system parameters in order to discover the "sweet spot" that achieves levitation and upward acceleration. These and other yet to be identified technical challenges need to be addressed via further empirical and theoretical studies.

V. Conclusion: The Way Forward

This report has reviewed and analyzed a number of antigravity concepts that are found within Newtonian gravity theory, General Relativity Theory, semi-classical quantum gravity theory, quantum field theory, and nonretarded quantum interatomic dispersion force theory. One found that plausible mechanisms exist within Newtonian and general relativistic theories whereby one could embody a realistic device that produces a significant antigravity force. However, one discovered that there are daunting technical challenges that arise in each of the proposed embodiments. Mechanical embodiments that produce antigravity forces require kilometer-sized apparatus, astronomical-sized masses and densities, or extreme mass velocities and accelerations. There are other subtleties involved, such as the possibility of different forms of matter having a highly nonlinear gravitational permeability, which could dramatically mitigate such large-scale requirements (see item 1 below for further discussion).

Negative energy has been produced in the lab in very small quantities. The technologies used for producing negative energy are nascent, and so it will be some time before it can be ascertained whether they are capable of producing the astronomical amounts of negative energy required to generate significant antigravity forces as discussed in Section III-C-2 (see item 2 below for further discussion).

Antigravity forces produced by quantum electromagnetic vacuum ZPF or by nonretarded quantum interatomic dispersion forces in a curved spacetime (that is, gravitational field) are very feeble, but there are proposals based on other theoretical and empirical studies which suggest that these forces can be amplified to macroscopic level. However, there are a number of difficult technical challenges to overcome in order to achieve success.

Going forward toward the demonstration of an antigravity generator will require the following steps to be taken:

- Antigravity via Dipole Gravitational Field Generators: Presently, the technology does not exist to achieve the astronomical mass densities, extreme velocities or accelerations of mass motion, and the large device dimensions required to produce large enough antigravity forces for useful propulsion. The issues are: 1) dense materials, and 2) gravitational properties of matter. Forward (Reference 14) suggests investigating neutron-neutron interactions. One could cool a gas of thermal neutrons from a nuclear reactor to extremely low temperatures using magnetic confinement or magneto-gravitational traps, and concentrate them into a small region through the interaction of the trap's magnetic field with the magnetic moment of the neutrons. The Fermi energy[12] of the bound neutrons limits the neutron density to $\sim 10^{3}$ kg/m^3. However, the formation of putative tetraneutrons[13] or the existence of a superconductive-type phase space condensation will create bosons that do not have this limitation. It turns out that exotic quantum states of matter such as Bose-Einstein (BE) and Fermionic condensates[14] transcend the Fermi energy limit and thus possess highly unusual material properties. BE condensates were first created in 1995 and Fermionic condensates were first created in 2003, but both are still undergoing laboratory exploration. As for the gravitational properties of matter, one knows from electromagnetism that the permeability (μ) of magnetic materials such as iron is anomalously large and nonlinear, which allows for the construction of highly efficient electromagnetic field generators. The gravitational equivalent to the magnetic permeability is a property of matter that is still largely unexplored. A material possessing an anomalously large, very nonlinear gravitational permeability (η) would be useful in the construction of highly efficient, very small scale gravitational field generators. One would expect all materials to have an η that is different from η_0 because the atoms comprising any material have quantum spin. Forward

[12] In condensed matter physics, this is the energy of the highest occupied quantum state in a system of fermions (that is, spin-½ particles such as electrons, nucleons, or atoms) at zero absolute temperature.
[13] A hypothetical stable cluster of four neutrons whereby recent empirical evidence suggests it exists. Readers should consult the technical literature for more information by using "tetraneutrons" as a search term.
[14] BE or Fermionic condensates are a macroscopic collection of bosons (spin-1 particles such as nucleons or atoms) or fermions that collapse into the same quantum state when they form at near-zero absolute temperature.

(Reference 14) reported that a rough estimate indicates there is a very small difference between η and η_0. It is thus necessary to implement a coordinated theoretical program to determine the value of η for all known forms of matter and an experimental program to find materials that might possess anomalously large or nonlinear properties that can be used to intensify time-varying gravitational fields. Forward (Reference 14) also described an unsuccessful experimental attempt to find materials that have the property of converting time-varying electromagnetic fields into time-varying gravitational fields. This speculative property exploits the fact that the magnetic and inertial moments are combined in an atom via the usual quantum angular and spin momentum coupling. Other theoretical and experimental concepts incorporating the use of rotating superconductors are reviewed by Hathaway (Reference 69). Note in particular that Hathaway reviews the emerging experimental observations of Martin Tajmar in which an apparent frame-dragging effect is observed near super-cooled rotating rings as measured by ring laser gyros and accelerometers. At the time of this writing these effects were being reported but not yet independently confirmed.

- Antigravity via Negative Energy: The assessment provided in Reference 70 concludes that small amounts of negative energy are already made in the lab, but one does not yet know there is access to larger amounts for extended periods of time over extended spatial distributions for the purpose of producing antigravity. In this regard, the following options for further exploration are proposed:

  – Squeezed quantum vacuum generators (see Appendix A): A dedicated research program to develop the two negative energy generator concepts described in Reference 70 will need to be established in order to evolve state-of-the-art quantum optics technology towards producing higher magnitudes of negative energy as well as special techniques required to separate out any positive energy fluxes that accompany the negative energy fluxes. Specifically, the Rabeau et al. (Reference 71,72) and Ries et al. (Reference 73) experimental programs should be followed as a template toward this goal. Quantum optics technology via high power fiber lasers, resonators, amplifier stages, beam conditioning stages, and so forth are rapidly advancing. So research should be conducted in parallel to invent additional ways to produce negative energy via innovative quantum optics.

  – Casimir effect: Even though the standard electromagnetic Casimir effect is feeble, and thus not likely to contribute to an antigravity engineering program, there are still a number of other electromagnetic and non-electromagnetic Casimir effects described in Appendix A that require further study. These other Casimir effects have not been explored with an eye toward testing them in the lab, and so there could be important new information yet to be discovered.

  – Moving Mirrors (a.k.a. the dynamical Casimir effect; see Appendix A): Even though this concept is too feeble to produce any useful flux of negative energy, the observable effects due to the change in the boundary conditions (for example, moving mirrors/cavity walls) of quantum fields provide crucial information on the quantum vacuum at the macroscopic level. Theoretical and laboratory efforts are underway to understand the dissipative effects of vacuum fluctuations (Reference 74,75). This dissipation mechanism should induce irradiation of photons, a phenomenon also known as the dynamical Casimir effect. This can be understood both as the creation of particles under nonadiabatic changes in the boundary conditions of quantum fields, or as classical parametric amplification with the zero-point energy of a vacuum field mode as an input state. More recent developments include models for the super-radiant amplification of photons with particular emphasis on its dynamics and the optimization of the involved parameters. Experimental concepts being pursued will try to reveal directly the presence of a non-empty vacuum by using a specifically designed device to amplify the virtual vacuum photons and produce real electromagnetic radiation via the parametric amplification of the vacuum fluctuations in an electromagnetic cavity. The 'amplifier' is a boundary undergoing an oscillation, and hence radiates energy due to the dissipative action against the vacuum photons. This line of investigation could serve as a very useful probe to explore the possibility of generating large fluxes of negative energy. One expects that a laboratory demonstration of the dynamical Casimir effect will occur before 2012.

- Dirac field states: As described in Section III-C-1, this involves either the superposition of two single particle electron states or the superposition of two multi-electron-positron states (Reference 41,42). This is still a nascent topic of study in quantum field theory. However, already a great deal of technology is dedicated to the manipulation and storage of electrons and positrons via solid state/condensed matter devices and particle accelerators. This research topic should be supported in order to establish how it could contribute to an experimental antigravity program.

- Quantum coherence effects: Other types of quantum coherence effects not already identified or invented should be theoretically developed and explored for the possibility of finding new free-field or interacting field configurations that produce a significant magnitude of negative energy which could be produced by technological means. [Note that Reference 70 showed that static, radially-dependent electric or magnetic fields and gravitationally squeezed vacuum electromagnetic zero-point fluctuations (see items 1 and 3 in Section III-C-1 and Appendix A) are not useful forms of negative energy.]

- Detecting Negative Energy in the Lab: Reference 70 identified proposals for observing negative energy in outer space and in the laboratory, but further work is needed to downscale astronomical techniques for use at the lab scale, and there is need to firm up our understanding of how lab detectors will respond to negative energy *in situ* if one is to exploit it for the production of antigravity forces. A first step in the latter direction was recently proposed by Marecki (Reference 76) who generalized the analysis of the output of balanced homodyne detectors (BHDs). The most important feature of these devices is their ability to quantify the quantum vacuum fluctuations of the electric field because the output of BHDs provides information on the one- and two-point functions of arbitrary states of quantum fields. Marecki computed the two-point function and the associated spectral density for the ground state of the quantum electric field in Casimir geometries, and predicts a position- and frequency-dependent pattern of BHD responses if a device of this type is placed inside a Casimir cavity. The proposed device allows for the direct detection of quantum vacuum fluctuations and provides a spatial mapping of the negative energy contained inside the cavity. This offers a potential new characterization of ground states in Casimir geometries, which would provide an understanding of the negative energy densities present in some regions in these geometries.

- Trapping and Storing Negative Energy: Ford and Roman (Reference 20) have only superficially addressed this topic, and there is very little technical literature that addresses it fully. A theoretical program to develop the physics and technology of trapping and storing negative energy will need to be supported, and such a program should be guided by the use of laboratory detectors such as the one proposed in the previous section. However, it is the opinion of the author that free-space negative energy sources appear to be a more desirable option for producing antigravity than stored negative energy.

- Antigravity via Quantum Vacuum Zero-Point Fluctuation Force: Calloni et al.'s experimental proposal reviewed in Section IV-A should be funded and performed by a high quality laboratory.
- Antigravity via Nonretarded Quantum Interatomic Dispersion Force: Pinto's experimental proposal reviewed in Section IV-B should be theoretically evaluated prior to funding an experiment. This proposal does contain enough rigor and credibility that it warrants a further look.

Appendix A

STATIC RADIAL ELECTRIC & MAGNETIC FIELDS

It is beyond the scope of this report to include all the technical configurations by which one can generate static, radially-dependent electric or magnetic fields. However, there remains the problem of engineering these fields to produce a borderline exotic energy state because classical electromagnetic theory states that every observer will see a non-negative energy density that is $\propto E^2 + B^2$, where the electric field (E) and magnetic field (B) strengths are measured in any observer's reference frame. It is not known how to increase the tension in these fields using current physics, but some new physics may provide an answer. This technical problem must be left for future investigation.

SQUEEZED QUANTUM VACUUM

Substantial theoretical and experimental work has shown that in many quantum systems the limits to measurement precision imposed by the quantum vacuum zero-point fluctuations (ZPF) can be breached by decreasing the noise in one observable (or measurable quantity) at the expense of increasing the noise in the conjugate observable; at the same time the variations in the first observable, say the energy, are reduced below the ZPF such that the energy becomes "negative." "Squeezing" is thus the control of quantum fluctuations and corresponding uncertainties, whereby one can squeeze/reduce the variance of one (physically important) observable quantity provided the variance in the (physically unimportant) conjugate variable is stretched/increased. The squeezed quantity possesses an unusually low variance, meaning less variance than would be expected on the basis of the equipartition theorem. One can in principle exploit quantum squeezing to extract energy from one place in the ordinary vacuum at the expense of accumulating excess energy elsewhere (Reference 21).

The squeezed state of the electromagnetic field is a primary example of a quantum field that has negative energy density and negative energy flux. Such a state became a physical reality in the laboratory as a result of the nonlinear-optics technique of "squeezing"—that is, of moving some of the quantum-fluctuations of laser light out of the cos[$\omega(t - z/c)$] part of the beam and into the sin[$\omega(t - z/c)$] part (Reference 77-82).[15] The observable that gets squeezed will have its fluctuations reduced below the vacuum ZPF. The act of squeezing transforms the phase space circular noise profile characteristic of the vacuum into an ellipse, whose semimajor and semiminor axes are given by unequal quadrature uncertainties (of the quantized electromagnetic field harmonic oscillator operators). This applies to coherent states in general, and the usual vacuum is also a coherent state with eigenvalue zero. As this ellipse rotates about the origin with angular frequency, ω, these unequal quadrature uncertainties manifest themselves in the electromagnetic field oscillator energy by periodic occurrences, which are separated by one quarter cycle, of both smaller and larger fluctuations compared to the unsqueezed vacuum.

Morris and Thorne (Reference 21) and Caves (Reference 83) point out that if one squeezes the vacuum—that is, if one puts vacuum rather than laser light into the input port of a squeezing device—then one gets at the output an electromagnetic field with

[15] ω is the angular frequency of light, t is time, and z denotes the z-axis direction of beam propagation.

weaker fluctuations and thus less energy density than the vacuum at locations where $\cos^2[\omega(t - z/c)] \cong 1$ and $\sin^2[\omega(t - z/c)] << 1$; but with greater fluctuations and thus greater energy density than the vacuum at locations where $\cos^2[\omega(t - z/c)] << 1$ and $\sin^2[\omega(t - z/c)] \cong 1$. Since the vacuum is defined to have vanishing energy density, any region with less energy density than the vacuum actually has a negative (renormalized) expectation value for the energy density. Therefore, a squeezed vacuum state consists of a traveling electromagnetic wave that oscillates back and forth between negative energy density and positive energy density, but has positive time-averaged energy density.

For the squeezed electromagnetic vacuum state, the energy density $\rho_{\text{E-sqvac}}$ is given by (Reference 84):

$$\rho_{\text{E-sqvac}} = \left(\frac{2\hbar\omega}{L^3}\right)\sinh\xi\left[\sinh\xi + \cosh\xi\cos\left(2\omega(t - z/c) + \delta\right)\right] \quad (\text{J}/\text{m}^3) \tag{A.1}$$

where L^3 is the volume of a large box with sides of length L (that is, put the quantum field in a box with periodic boundary conditions), ξ is the squeezed state amplitude (giving a measure of the mean photon number in a squeezed state), and δ is the phase of squeezing. Equation (A.1) shows that $\rho_{\text{E-sqvac}}$ falls below zero once every cycle when the condition $\cosh\xi > \sinh\xi$ is met. It turns out that this is always true for every nonzero value of ξ, so $\rho_{\text{E-sqvac}}$ becomes negative at some point in the cycle for a general squeezed vacuum state. On another note, when a quantum state is close to a squeezed vacuum state, there will almost always be some negative energy densities present.

GRAVITATIONALLY SQUEEZED ELECTROMAGNETIC ZERO-POINT FLUCTUATIONS

A natural source of negative energy comes from the effect that gravitational fields (of astronomical bodies) in space have upon the surrounding quantum vacuum. For example, the gravitational field of the Earth produces a zone of negative energy around it by dragging some of the virtual quanta (aka vacuum ZPF) downward. This concept was initially developed in the 1970s as a byproduct of studies on quantum field theory in curved space (Reference 37). However, Hochberg and Kephart (Reference 33) derived an important application of this concept to the problem of creating and stabilizing traversable wormholes. They showed that one can utilize the negative energy densities, which arise from distortion of the vacuum ZPF due to the interaction with a prescribed gravitational background, for providing a violation of the energy conditions. The squeezed quantum states of quantum optics provide a natural form of matter having negative energy density. The corresponding local vacuum state energy density is $\rho_{\text{E-gsvac}} = -2\pi^2\hbar c/\lambda^4$, where λ is the ZPF mode wavelength under consideration in the gravitational squeezing effect (Reference 70).

The analysis, via quantum optics, showed that gravitation itself provides the mechanism for generating the squeezed vacuum states needed to support stable traversable wormholes. The production of negative energy densities via a squeezed vacuum is a necessary and unavoidable consequence of the interaction or coupling between ordinary matter and gravity, and this defines what is meant by gravitationally squeezed vacuum states. One is presently unaware of any way to artificially produce gravitational squeezing of the vacuum in the laboratory for the purpose of inducing an antigravity effect for propulsion applications.

QUANTUM VACUUM FIELD STRESS: NEGATIVE ENERGY FROM THE CASIMIR EFFECT

The Casimir effect is by far the easiest and most well known way to generate negative energy in the lab. The Casimir effect that is familiar to most people is the force that is associated with the electromagnetic quantum vacuum (Reference 85). This is an attractive force that must exist between any two neutral (uncharged), parallel, flat, conducting surfaces (for example, metallic plates) in a vacuum. This force has been well measured and it can be attributed to a minute imbalance in the vacuum electromagnetic zero-point energy density inside the cavity between the conducting surfaces versus the vacuum electromagnetic zero-point energy density in the free-space region outside of the cavity (Reference 86-88). See Figure 4 for an illustration of this effect.

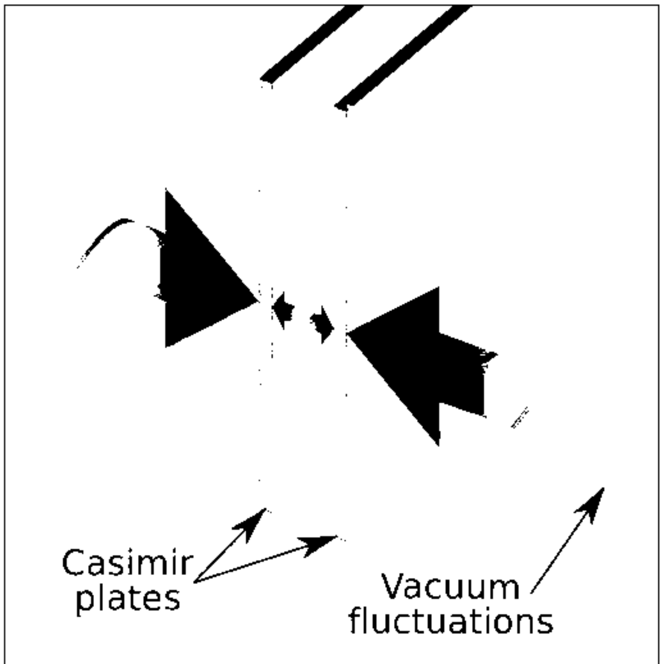

Figure 4. Illustration of the Casimir Effect

It turns out that there are many different types of Casimir effects found in quantum field theory (Reference 34-36,40,89). For example, if one introduces a single infinite plane conductor into the Minkowski (flat spacetime) vacuum by bringing it adiabatically from infinity so that whatever quantum fields are present suffer no excitation but remain in their ground states, then the vacuum (electromagnetic) stresses induced by the presence of the infinite plane conductor produces a Casimir effect. This result holds equally well when two parallel plane conductors (with separation distance *d*) are present, which gives rise to the familiar Casimir effect inside a cavity. Note that in both cases, the spacetime manifold is made incomplete by the introduction of the plane conductor boundary condition(s). The vacuum region put under stress by the presence of the plane conductor(s) is called the Casimir vacuum. The generic expression for the energy density of the Casimir effect is $\rho_{CE} = -A\hbar c/d^4$, where $A = \zeta(D)/8\pi^2$ in spacetimes of arbitrary dimension D (Reference 34-36). The appearance of the zeta-function $\zeta(D)$ is characteristic of expressions for vacuum stress-energy tensors, $T^{\mu\nu}_{vac}$. In our familiar 4-dimensional spacetime (D = 4) the equation exists $A = \pi^2/720$. To calculate $T^{\mu\nu}_{vac}$ for a given quantum field is to calculate its associated Casimir effect.

Analogs of the Casimir effect also exist for fields other than the electromagnetic field. When considering the vacuum state of other fields, one must consider boundary conditions that are analogous to the perfect-conductor boundary conditions for the electromagnetic field at the surfaces of the plates (Reference 34-36,40). Other fields are not electromagnetic in nature; that is to say they are non-Maxwellian, and so the perfect-conductor boundary conditions do not apply to them. It turns out that complete manifolds exhibit what is called the topological Casimir effect for any non-Maxwellian fields. In order to define boundary conditions for other fields one replaces the conductor boundary conditions and Minkowski spacetime by a manifold of the form $\Re \times \Sigma$ (that is, a product space), where $\Re$ is the real line defining the time dimension for this particular product space and Σ is a flat 3-dimensional manifold having any one of the following topologies: $\Re^2 \times S^1$, $\Re \times T^2$, T^3, $\Re \times K^2$, and so forth, $\Re$ being the real line that defines any linear space dimension (for example, $\Re$ = line, $\Re^2$ = 2-dimensional plane), T^n being the n-torus, K^2 the 2-dimensional Klein bottle, S^1 the circle, and so forth.

The case $\Sigma = \Re^2 \times S^1$ has the closest resemblance to the electromagnetic Casimir effect, the difference being that instead of imposing conductor boundary conditions, one imposes periodic boundary conditions on some of the space coordinates in the 3-dimensional manifold. When imposing this topological constraint on the field theoretic calculation of the topological Casimir effect (for linear massless fields), one finds that the generic expression for the energy density is also $\rho_{CE} = -A\hbar c / d^4$, where $A = \pm d_f\left(\pi^2 / 90\right)$, d_f is the number of degrees of freedom (for example, helicity states) per spatial point, the plus sign holds for boson fields (giving a negative energy density) and the negative sign for fermion fields (giving a positive energy density).

If one were to admit spin structure in the manifolds described above and the field is spinorial, then there is another important subtlety that must be taken into account when evaluating $T^{\mu\nu}_{vac}$. However, this introduces an additional complexity involving the relationship between the spin structure and the global structure (that is, the configuration space or fiber bundle) of the field in question whereby the topology not only of the base manifold, but of the fiber bundle itself has an effect on $T^{\mu\nu}_{vac}$. In addition to this, there are (compactified) extra-space dimensional quantum field (that is, D-Brane or "brane world") analogs of the Casimir effect yet to be explored. But a detailed consideration of these is beyond the scope of this report and will be left for future investigation.

As a final note, one points out that the methods used to obtain the electromagnetic $T^{\mu\nu}_{vac}$ between parallel plane conductors can also be used when the conductors are not parallel but are joined together along a line of intersection. If the conductors have curved surfaces instead, then one obtains results that are similar to the case of intersecting conductors. These geometries have also been evaluated for the case of dielectric media. These particular cases will not be considered further since there are technical subtleties involved that complicate the calculations and application of the different approaches. This topic will also be left for future investigation.

DYNAMICAL CASIMIR EFFECT: MOVING MIRRORS

Negative energy can be created by a single moving reflecting (conducting) surface (a.k.a. a moving mirror). A mirror moving with increasing acceleration generates a flux of negative energy that emanates from its surface and flows out into the space ahead of the mirror (Reference 37,90). See Figure 5 (below) for an illustration of this effect. This is essentially the simple case of an infinite plane conductor undergoing acceleration perpendicular to its surface. If the acceleration varies with time, the conductor will generally emit or absorb photons (that is, exchange energy with the vacuum), even though it is neutral. This is an example of the well-known quantum phenomenon of parametric excitation. The parameters of the electromagnetic field oscillators (for example, their frequency distribution function) change with time owing to the acceleration of the mirror (Reference 91). This effect is known to be exceedingly small. However, recent theoretical and technological developments suggest that laboratory investigations of the dynamical Casimir effect will begin in the very near future (Reference 74,75).

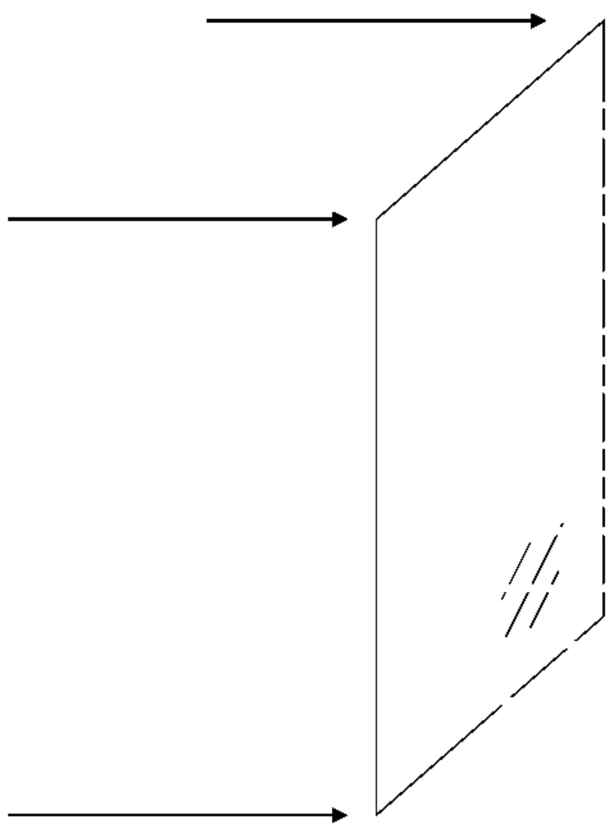

Figure 5. Negative Energy Flux (Gold) Emanating From a Moving Mirror

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